An Introduction to Secure Multi-Party Computation

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Overview

Introduction: What is Secure Multiparty Computation

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- Computation: a known function is evaluated

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- Secure: each party's input remains secret

MPC allows a set of parties to joinly compute a function on their secret inputs.

Introduction: An Example (Yao 1982)

Two millionaires want to know who has more money without revealing their assets.

- Parties: two millionaires Alice and Bob
- Function: $X_A > X_B$?
- Inputs: assets X_A and X_B

Trivial solution: a trusted third party.
It gets X_A and X_B and announces who is the richer.

Introduction: Trusted Third Party

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To compute on private data there are two main solutions:

- Homomorphic encryption
- Multiparty Computation

Introduction: Formalization of the Model

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These instructions can be local computations and exchanges of data.
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Formally:

- Parties: $P_1, \ldots, P_n$
- Inputs: $x_1, \ldots, x_n$
- Public function: f
- Output: $y = f(x_1, \ldots, x_n)$

Introduction: Security Properties

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- Correctness: depending on the MPC protocol, even if a subset of parties are colluding sharing information or deviating by the protocol, they should not be able to force honest parties to output an incorrect result.

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Introduction: Types of Security

MPC protocols can differ in the type of security guaranteed. There are 3 main types:

- Passive security: security properties are guaranteed with semi-honest (or "honest but curious“) adversaries, which follow the rules of the protocol but wanting to extract more information from their observed data.

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- Covert security: between passive and active security. Quite informal definition: “secure enough”.

Introduction: Boolean MPC and Arithmetic MPC

There are two main classes of MPC protocols:

- Boolean MPC: the function is represented by a boolean circuit. There are two parties, the circuit constructor A and the circuit evaluator B. A encrypts or garbles the circuit and sends it to B that evaluates it with his input and learns the output. This protocol is called *Garbled Circuit*.

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- Arithmetic MPC: inputs are shared using some secret sharing scheme, then the function is computed on this sharings, using addition, multiplication and other arithmetic operations.

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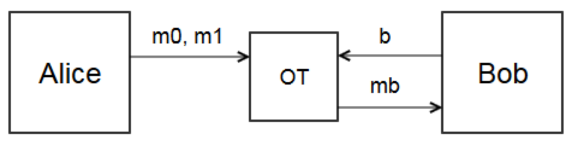
Ingredients:

- Double key simmetric encryption: given a plaintext m and two keys k_1, k_2, we denote by $E_{k_1,k_2}(m)$ the encryption of m with keys k_1, k_2. For example we can use $E_{k_1,k_2}(m) = AES_{k_1}(AES_{k_2}(m))$.
  In order to check the validity of a plaintext we can add some redundancy.
- Oblivious Transfer (OT)

Boolean MPC: Oblivious Transfer



An oblivious transfer protocol is an interactive protocol between two parties: a Sender and a Receiver, each providing some inputs.

- The Sender inputs a couple of bits (or numbers) m_0, m_1

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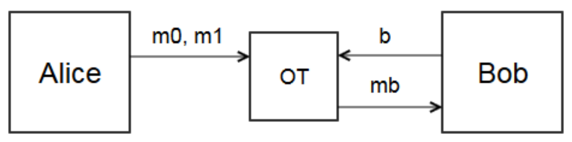

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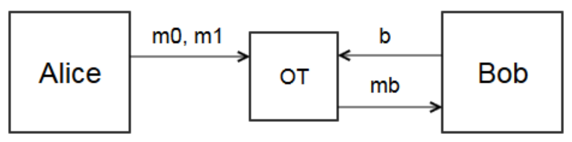

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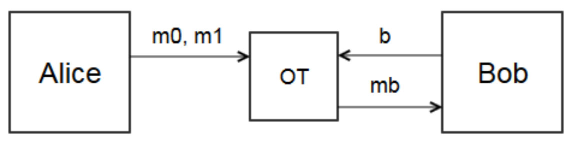

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- The Sender inputs a couple of bits (or numbers) m_0, m_1
- The input of the Receiver is a single bit b.
- At the end of the protocol, the Receiver gets the value m_b
- The Sender knows nothing about b and the Receiver knows nothing about the other value m_{1-b}

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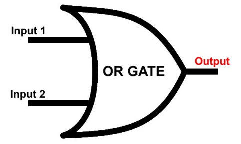
- For every output wire, Alice announces the correspondence between w_j^i and i so Bob can learn the output.
- Alice sends the garbled values of every truth table to Bob.

Boolean MPC: Example of Garbled Truth Table

OR gate



W_0	W_1	W
0	0	0
0	1	1
1	0	1
1	1	1

W_0	W_1	W	Garbled value
w_0^0	w_1^0	w^0	$\mathbb{E}_{w_0^0, w_1^0}(w^0)$
w_0^0	w_1^1	w^1	$\mathbb{E}_{w_0^0, w_1^1}(w^1)$
w_0^1	w_1^0	w^1	$\mathbb{E}_{w_0^1, w_1^0}(w^1)$
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Boolean MPC: Evaluating the Circuit

- Using an oblivious transfer, Bob asks Alice the garbled values for its secret inputs.

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- Starting from the input wires, for each gate Bob decrypts the values in the garbled truth table, finding the valid plaintext related to the output wire of the gate.

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Boolean MPC: Evaluating the Circuit

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- Bob continues with the next gate.
- When Bob has a garbled value Z for an output wire, he learns the output bit looking at values (Z_0, Z_1) and checking if $Z = Z_0$ or $Z = Z_1$.

Boolean MPC: Example of Evaluation of a Gate

- Gate values: X_1, X_2, X_3, X_4.
- Bob has the two garbled values of the inputs of this gate: w_1, w_2.
- He computes $D_{w_1,w_2}(X_i)$ for each i.
- When he finds a valid plaintext, he gets the garbled value associated to the output wire of this gate.

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Arithmetic MPC

If the function to be evaluated can be easily expressed in an arithmetic form, then it is convenient to use an MPC protocol based on *secret sharing*. This kind of protocols usually works on finite algebraic structures (finite fields or rings).

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There are a lot of secret sharing schemes, the most used are:

- Additive secret sharing;
- Shamir secret sharing.

Arithmetic MPC: Additive Secret Sharing

Suppose n players $P_1, \ldots, P_n$.
If a player P wants to share its secret input x, he randomly generates n shares $x^{(j)}$ such that

$$x = \sum_{j=1}^{n} x^{(j)}$$

Then P sends $x^{(j)}$ to player P_j.
The shared value of x is denoted as

$$[[x]] = (x^{(1)}, \ldots, x^{(n)})$$

This means that every player has a little part of x but nobody knows the actual value.

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Arithmetic MPC: Example of Additive Secret Sharing

Suppose 5 players $P_1, \dots, P_5$. A dealer wants to share the secret $s = 6 \in \mathbb{F}_{11}$.

- He generates 4 random elements of $\mathbb{F}_{11}$

$$s^{(1)} = 5, s^{(2)} = 3, s^{(3)} = 8, s^{(4)} = 0$$

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$$s^{(5)} = s - \sum_{i=1}^{4} s^{(i)} = 6 - 5 = 1$$

and distributes $s^{(i)}$ to P_i.

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  and distributes $s^{(i)}$ to P_i.
- All players can reconstruct the secret sharing their values to get

$$s = \sum_{i=1}^{5} s^{(i)}$$

Arithmetic MPC: Shamir Secret Sharing Scheme

Suppose n players $P_1, \ldots, P_n$ and $t \leq n$. A secret x of the player P can be shared as follow:

- P secretly chooses a random polynomial f of degree $t-1$ such that $f(0) = x$.

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The shared value of x is denoted as

$$[[x]] = ((1, f(1)), \ldots, (n, f(n)))$$

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Arithmetic MPC: Lagrange Interpolation

Given a set of points $\{(x_1, y_1), \ldots, (x_r, y_r)\}$ such that $x_i \neq x_j$ for every $i \neq j$, then exists a unique polinomial f of degree $\leq r - 1$ such that $f(x_i) = y_i$ for each i.

The polynomial f can be constructed as follows:

- Define
$$\delta_i(x) = \frac{\prod_{\substack{j=1 \\ j \neq i}}^{r} (x - x_j)}{\prod_{\substack{j=1 \\ j \neq i}}^{r} (x_i - x_j)}$$
we see that for each i $\delta_i(x_i) = 1$ and $\delta_i(x_j) = 0$ if $i \neq j$.
- Then set
$$f(x) = \sum_{i=1}^{r} \delta_i(x) y_i$$

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Arithmetic MPC: Example of Shamir Secret Sharing

Suppose 5 players $P_1, \ldots, P_5$, we work on $\mathbb{F}_{11}$. A dealer want to share the secret $s = 3$ with the threshold $t = 3$.

- He chooses a random polynomial such that $f(0) = s = 3$:

$$f(x) = 5x^2 + 10x + 3$$

- He gives $(i, f(i))$ to the i-th player:

$$(1,7),(2,10),(3,1),(4,2),(5,2)$$

- If P_1, P_4, P_5 want to learn the secret, they use Lagrange interpolation and find f and then $s = 3$.

Arithmetic MPC: Arithmetic with Additive Secret Sharing

Suppose n players have $[[x]] = (x^{(i)})_{i=1}^{n}$ and $[[y]] = (y^{(i)})_{i=1}^{n}$. They want to compute the shared value of the sum: $[[z]] = [[x + y]]$

- Each player P_i sets $z^{(i)} = x^{(i)} + y^{(i)}$.
- In fact:

$$z = \sum_{i=1}^{n}(x^{(i)} + y^{(i)}) = \sum_{i=1}^{n} x^{(i)} + \sum_{i=1}^{n} y^{(i)} = x + y$$

- This is an operation without communication.

Arithmetic MPC: Arithmetic with Additive Secret Sharing

If players have $[[x]]$ and they want to compute $[[z]] = [[cx]]$ for any public c:

- Each player P_i sets $z^{(i)} = cx^{(i)}$
- In fact:

$$z = \sum_{i=1}^{n} cx^{(i)} = c \sum_{i=1}^{n} x^{(i)} = cx$$

- This is another communication-free operation.

Arithmetic MPC: Arithmetic with Additive Secret Sharing

To perform a multiplication there are some different methods. For example the SPDZ protocol uses the Beaver's trick with some precomputed "multiplication triples":

$$([[a]], [[b]], [[c]]) \quad \text{such that} \quad ab = c$$

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How those triples are generated depends on the MPC protocol used, SPDZ bases his triple generation on homomorphic encryption.

Arithmetic MPC: Arithmetic with Additive Secret Sharing

Suppose parties have $[[x]]$ and $[[y]]$.
To compute $[[z]] = [[xy]]$:

- Players compute $[[\rho]] = [[x]] - [[a]]$ and reveal ρ

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- The output is $[[z]] = [[c]] + [[\rho b]] + [[\sigma a]] + \rho\sigma$

Arithmetic MPC: Arithmetic with Additive Secret Sharing

Summarizing:

- additions and scalar multiplications are "free" operations in terms of communication
- performing a multiplication costs 1 round of communication
- The complexity of a function to be evaluated in MPC is linked to the number of multiplications

Arithmetic MPC: A Simple Example (1)

Two parties, P_1 and P_2 want to compute $f(x_1, x_2) = x_1 x_2 + x_1$ in $\mathbb{F}_7$. Suppose $x_1 = 2$ and $x_2 = 5$.

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- They share their inputs. P_1 generates a random $x_1^{(1)} = 3$ and sets $x_1^{(2)} = 2 - 3 = 6$. P_1 sends $x_1^{(2)}$ to P_2, then we have:

$$[[x_1]] = [[2]] = (3, 6)$$

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- P_2 does the same, he generates $x_2^{(1)} = 1$ and sets $x_2^{(2)} = 5 - 1 = 4$ and sends $x_2^{(1)}$ to P_1. Then

$$[[x_2]] = [[5]] = (1, 4)$$

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Arithmetic MPC: A Simple Example (2)

Now they want to compute $[[x_1 x_2]]$. They pick a precomputed multiplication triple:

$$([[a]], [[b]], [[c]]) = ([[2]], [[6]], [[5]])$$

such that:

$$[[2]] = (1, 1), \ [[6]] = (4, 2), \ [[5]] = (0, 5)$$

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Arithmetic MPC: A Simple Example (3)

Multiplication subprotocol:

- P_1 computes

$$\rho^{(1)} = x_1^{(1)} - a^{(1)} = 2$$
$$\sigma^{(1)} = x_2^{(1)} - b^{(1)} = 4$$

Arithmetic MPC: A Simple Example (3)

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- P_1 sets $z^{(1)} = c^{(1)} + \rho b^{(1)} + \sigma a^{(1)} + \rho\sigma = 6$

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Arithmetic MPC: A Simple Example (4)

Now we have $[[z]] = (6, 4) = [[3]] = [[2 \cdot 5]]$. To obtain the output of $f(2, 5)$ we need to compute $[[z + x_1]]$.

- P_1 sets $w^{(1)} = z^{(1)} + x_1^{(1)} = 2$

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- P_1 sets $w^{(1)} = z^{(1)} + x_1^{(1)} = 2$
- P_2 sets $w^{(2)} = z^{(2)} + x_1^{(2)} = 3$
- Now they excange their shares and learn the output $w = 2 + 3 = 5$, in fact $f(2, 5) = 5$.

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Arithmetic MPC: Offline Phase vs Online Phase

Some protocols split computation in two parts:

- A preprocessing phase that depends on the function and is independent on the inputs. It is called "offline phase".
- An evaluation phase: players uses their inputs and compute the function, this is called "online phase".

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For example in the SPDZ protocol, the offline phase is represented by the triples generation, while the online phase by the actual computation of the function f.

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Active Security: How to Prevent Active Attacks?

How to deal with malicious adversaries that can deviate from the protocol? When the protocol says "send x " they could send y or some crafted values.

There are some solutions, we see how the SPDZ protocol solves this problem.

Active Security: MAC Keys

Each player P_i generates a MAC key $\Delta^{(i)}$. We define

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Now shares of the value $x \in \mathbb{F}$ are of the form

$$[[x]] = (\underbrace{x^{(1)}, \ldots, x^{(n)}}_{\text{shares}}, \underbrace{m(x)^{(1)}, \ldots, m(x)^{(n)}}_{\text{MAC shares}}, \underbrace{\Delta^{(1)}, \ldots, \Delta^{(n)}}_{\text{MAC keys}})$$

Such that:

$$x = \sum_{i=1}^{n} x^{(i)}, \qquad x \cdot \Delta = \sum_{i=1}^{n} m(x)^{(i)}$$

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Active Security: MAC Keys

If a malicious player sends the wrong values for $x^{(i)}$, he can't modify his MAC shares $m(x)^{(i)}$ to be consistent with the new value since he has not other MAC shares and Δ.

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When the function is evaluated and players hold the shared output, before revealing it to all parties, there is a general MAC check on all the values opened during the protocol.
If this check passes, then the output is revealed and accepted.

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Real-world Applications

Since 2008 there were a lot of real-world applications of MPC, for example:

- Danish sugar beet auction
- Benchmarking
- Satellite collisions
- Machine learning on private data

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Libraries

There are a lot of libraries that implement some MPC functionalities. Some examples:

- SCALE-MAMBA
- MP-SPDZ
- libSCAPI
- Fresco
- ...and many other

References

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Thank you.
Questions?

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